ChE-304 Problem Set 2

Week 2

Problem 1

Should I heat my house with a Carnot heat pump or an electric heater? Electric heaters are very efficient. ~100% of the electrical work ends up as heat in the house.
I would like to heat my house to 24°C and the temperature outside in the winter is, on average -4°C.

Solution:

The efficiency is: $COP_W = \dfrac{Q_H}{W} = \dfrac{T_H}{T_H - T_C} = \dfrac{273+24}{273+24-(273-4)} = 10.6$

For an electric heater $COP_W = \dfrac{Q_H}{W} \approx 1$

A Carnot heat pump is about 10 times more efficient! Definitely use the heat pump.

Problem 2
Let's calculate the efficiency of a typical refrigerator/freezer in your home. A typical refrigeration fluid is tetrafluoroethane (R134a). The evaporator usually runs at atmospheric pressure and R134a boils at -26°C. Even on hot summer days, which can reach on average 29°C in Switzerland, the refrigerator condenser fluid should always remain at least 10°C hotter than the environment.

Assume there is 1 kg of refrigerant. You can also assume that all the work given by the compressor is converted to enthalpy in the fluid. Finally, assume that the expansion valve is an isentropic process. Use the data in the annex.
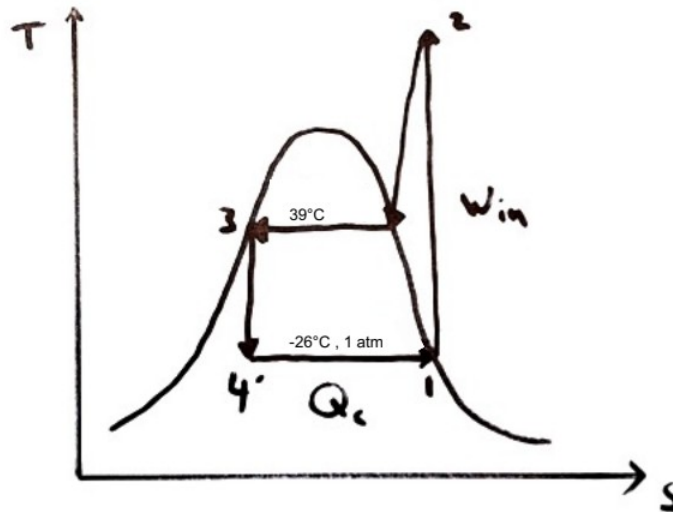
What is the efficiency?

Assuming that a refrigerator uses about 500 Watt, what is its cooling power and how does that compare to a Carnot refrigerator?

Hint: look at figure 1.12. Find the data points on the graph at which we need to determine the properties. You can find out these properties by what you know about the Rankine cycle.

Solution
In the solution, we will refer to the points on Fig. 1.12. We can sketch what we know:

We know when it boils and at what T and P from the saturation data (-26°C at 1 atm). We also know that the condensing temperature at high P has to be at 39°C to stay 10°C above 29°.

Point 1 on Fig. 1.12 has to be at boiling temperature and atmospheric pressure but has to be 100% gas. At this point we have T=-26°C, P=1 atm and 100% gas (from the annex): $S_1=S_{gas}=1.7471$ J/(g K)
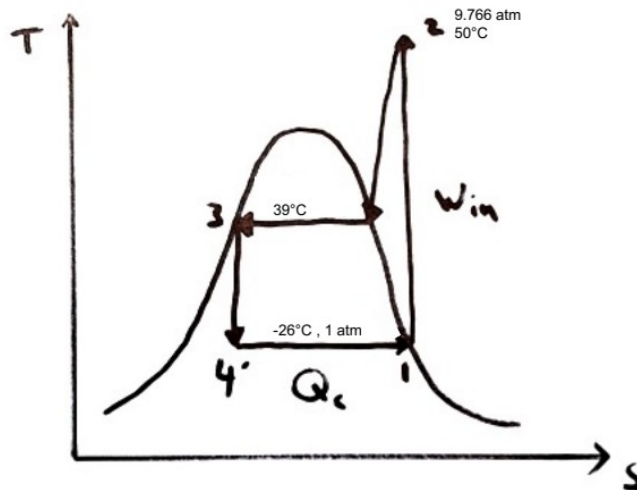
$H_1 = H_{gas} = 382.82$ kJ/(Kg)

To calculate efficiency, we have:

$$COP_{Refrigeration} = \frac{|q_c|}{|W|} = \dot{\iota} \frac{h_1 - h_{4'}}{h_2 - h_1}$$ (note that we will use point 4' on fig. 1.12 because we assume an isentropic (cst S) change)

We need to determine enthalpy for points 2 and 4'. For point 2, we know it has the same entropy as 1 (the stream gets to 2 by an isentropic change). However, we don't know what temperature or pressure it is at. We know that it has to be at the same pressure as 2' (because points 3, 2' and 2 are all at the same pressure).

We know the liquid condenses at 39°C. Therefore, we need to look up the saturation data (and especially the saturation pressure at 39°C). According to the annex, the pressure is at 9.766 atm!

We therefore know the pressure (9.766 atm) and entropy (1.7471 J/(g K)) of point 2. Let's try to find the temperature and enthalpy. Fortunately, we have this data in the Annex.

The entropy at 50°C and 9.766 atm is very close (1.7495 J/(g K)) so let's take this point.

The enthalpy is:

H_2= 431.10 kJ/kg

We now only need to know the enthalpy at point 4'. Here we need to proceed similarly as for 2. We can find the entropy of point 3 and assume an isentropic change to find point 4'.

Point 3 is the liquid equivalent of point 2' (Same T and P). Therefore, we already know the pressure and temperature (39°C and 9.766 atm). The entropy this point is:

$S_3 = S_{liq,39°C,9.766\ atm} = 1.1858\ J/(g\ K) = S_{4'}$

So we know the entropy of $S_{4'}$ and we know it's on the boiling line (same T and P as 1: T=-26°C, P=1 atm) but we don't know what proportion is a liquid and what is a gas. We will call x the liquid mass fraction of R134a at point 4'

We know the entropy of pure gas ($S_{4',liq}$) and pure liquid ($S_{4',liq}$) for the conditions at 4'. Therefore:

$$S_{4'} = x\,S_{4',liq} + (1-x)\,S_{4',gas}$$
$$S_{4'} - S_{4',gas} = x\left(S¿¿4',liq - S_{4',gas}\right)¿$$
$$x = \frac{S_{4'} - S_{4',gas}}{S_{4',liq} - S_{4',gas}} = \frac{1.1858 - 1.7471}{0.86943 - 1.7471} = 0.64$$

→ $H_{4'} = x\,H_{4',liq} + (1-x)\,H_{4',gas} = 0.64 \times 165.9 + 0.36 \times 382.82 = 244\ KJ/kg$

The efficiency is: $COP_{Refrigeration} = \frac{|q_c|}{|W|} = ¿\ \frac{h_1 - h_{4'}}{h_2 - h_1} = \frac{383 - 244}{431 - 383} = 2.9$

It's cooling power is 2.9 *500 W = 1448 W

For a Carnot fridge it would be 500 W $\times \dfrac{T_C}{T_H - T_C} = 500 \times \dfrac{273 - 26}{39 + 26} = 500 \times 3.8 = 1900\ W$

Annex: Saturation properties   Data pulled for 1,1,1,2-tetrafluoro-Ethane, (R134a) from:

http://webbook.nist.gov/

Liquid Phase Data

Data on Saturation Curve

Temperature (C)	Pressure (atm)	Density (mol/l)	Volume (l/mol)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (F/atm)	Viscosity (uPa*s)	Therm. Cond. (W/m*K)	Surf. Tension (N/m)	Phase
-31.000	0.79392	13.637	0.073332	159.46	159.52	0.84339	0.84264	1.2710	764.94	-0.049807	406.92	0.10624	0.016194	liquid
-26.000	1.0034	13.490	0.074127	165.83	165.90	0.86943	0.84825	1.2807	741.63	-0.047421	378.30	0.10387	0.015425	liquid
-21.000	1.2543	13.342	0.074951	172.25	172.34	0.89514	0.85396	1.2909	718.42	-0.044825	352.46	0.10153	0.014664	liquid
-16.000	1.5522	13.191	0.075807	178.72	178.83	0.92054	0.85977	1.3017	695.29	-0.041991	329.00	0.099225	0.013912	liquid
-11.000	1.9028	13.038	0.076699	185.24	185.38	0.94566	0.86569	1.3132	672.24	-0.038887	307.61	0.096944	0.013167	liquid
-6.0000	2.3121	12.882	0.077630	191.81	191.99	0.97051	0.87171	1.3254	649.22	-0.035475	288.00	0.094689	0.012431	liquid
-1.0000	2.7865	12.722	0.078603	198.44	198.66	0.99511	0.87785	1.3383	626.24	-0.031707	269.96	0.092457	0.011704	liquid
4.0000	3.3325	12.559	0.079623	205.13	205.40	1.0195	0.88410	1.3523	603.27	-0.027530	253.29	0.090246	0.010986	liquid
9.0000	3.9569	12.392	0.080695	211.89	212.21	1.0437	0.89048	1.3672	580.29	-0.022877	237.83	0.088054	0.010278	liquid
14.000	4.6670	12.221	0.081825	218.71	219.09	1.0677	0.89700	1.3835	557.28	-0.017667	223.42	0.085878	0.0095797	liquid
19.000	5.4699	12.045	0.083021	225.61	226.06	1.0915	0.90366	1.4011	534.23	-0.011801	209.96	0.083715	0.0088918	liquid
24.000	6.3734	11.864	0.084290	232.59	233.12	1.1152	0.91048	1.4205	511.10	-0.0051558	197.32	0.081564	0.0082146	liquid
29.000	7.3852	11.676	0.085643	239.65	240.28	1.1388	0.91748	1.4419	487.90	0.0024230	185.42	0.079420	0.0075488	liquid
34.000	8.5135	11.482	0.087091	246.80	247.54	1.1623	0.92468	1.4658	464.58	0.011131	174.18	0.077281	0.0068949	liquid
39.000	9.7666	11.280	0.088649	254.06	254.92	1.1858	0.93212	1.4926	441.13	0.021221	163.52	0.075143	0.0062535	liquid
44.000	11.153	11.070	0.090336	261.42	262.43	1.2092	0.93981	1.5232	417.53	0.033025	153.37	0.073003	0.0056255	liquid
49.000	12.683	10.849	0.092174	268.91	270.07	1.2327	0.94781	1.5584	393.74	0.046984	143.66	0.070857	0.0050116	liquid

Vapor Phase Data

Data on Saturation Curve

Temperature (C)	Pressure (atm)	Density (mol/l)	Volume (l/mol)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (F/atm)	Viscosity (uPa*s)	Therm. Cond. (W/m*K)	Phase
-31.000	0.79392	0.041473	24.112	360.68	379.69	1.7526	0.67550	0.77755	145.04	9.0236	9.5996	0.0089085	vapor
-26.000	1.0034	0.051698	19.343	363.55	382.82	1.7471	0.68859	0.79444	145.67	8.3821	9.7818	0.0093196	vapor
-21.000	1.2543	0.063820	15.669	366.42	385.94	1.7422	0.70194	0.81215	146.19	7.8137	9.9633	0.0097333	vapor
-16.000	1.5522	0.078086	12.806	369.28	389.02	1.7379	0.71555	0.83075	146.59	7.3070	10.144	0.010150	vapor
-11.000	1.9028	0.094762	10.553	372.12	392.06	1.7341	0.72941	0.85031	146.85	6.8534	10.325	0.010571	vapor
-6.0000	2.3121	0.11414	8.7612	374.95	395.06	1.7307	0.74354	0.87094	146.98	6.4458	10.507	0.010996	vapor
-1.0000	2.7865	0.13654	7.3241	377.75	398.02	1.7276	0.75792	0.89272	146.96	6.0788	10.689	0.011427	vapor
4.0000	3.3325	0.16231	6.1612	380.53	400.92	1.7250	0.77254	0.91581	146.79	5.7479	10.874	0.011865	vapor
9.0000	3.9569	0.19183	5.2128	383.27	403.76	1.7226	0.78742	0.94036	146.47	5.4491	11.061	0.012312	vapor
14.000	4.6670	0.22556	4.4334	385.98	406.53	1.7204	0.80254	0.96659	145.97	5.1794	11.252	0.012769	vapor
19.000	5.4699	0.26398	3.7882	388.64	409.22	1.7184	0.81792	0.99476	145.31	4.9360	11.448	0.013239	vapor
24.000	6.3734	0.30764	3.2506	391.25	411.82	1.7166	0.83357	1.0252	144.45	4.7166	11.651	0.013725	vapor
29.000	7.3852	0.35720	2.7996	393.80	414.33	1.7148	0.84951	1.0585	143.40	4.5193	11.863	0.014232	vapor
34.000	8.5135	0.41340	2.4190	396.27	416.72	1.7131	0.86577	1.0950	142.14	4.3422	12.086	0.014764	vapor
39.000	9.7666	0.47713	2.0958	398.66	418.99	1.7114	0.88238	1.1358	140.66	4.1839	12.323	0.015329	vapor
44.000	11.153	0.54948	1.8199	400.96	421.11	1.7096	0.89941	1.1818	138.95	4.0430	12.579	0.015935	vapor
49.000	12.683	0.63174	1.5829	403.13	423.07	1.7077	0.91692	1.2345	136.98	3.9184	12.858	0.016595	vapor

Isobaric Data for P = 9.7660 atm

Temperature (C)	Pressure (atm)	Density (mol/l)	Volume (l/mol)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (F/atm)	Viscosity (uPa*s)	Therm. Cond. (W/m*K)	Phase
20.000	9.7660	12.032	0.083113	226.70	227.50	1.0952	0.90475	1.4007	534.10	-0.011844	208.89	0.083603	liquid
25.000	9.7660	11.846	0.084417	233.74	234.56	1.1191	0.91161	1.4208	510.20	-0.0049433	196.08	0.081397	liquid
30.000	9.7660	11.653	0.085818	240.88	241.72	1.1429	0.91869	1.4434	485.95	0.0030563	183.95	0.079181	liquid
35.000	9.7660	11.450	0.087336	248.15	249.00	1.1667	0.92603	1.4691	461.27	0.012445	172.40	0.076949	liquid
38.998	9.7660	11.280	0.088649	254.06	254.92	1.1858	0.93211	1.4926	441.14	0.021216	163.52	0.075144	liquid
38.998	9.7660	0.47710	2.0960	398.66	418.99	1.7114	0.88237	1.1358	140.66	4.1840	12.323	0.015328	vapor
40.000	9.7660	0.47367	2.1112	399.65	420.13	1.7150	0.88028	1.1276	141.31	4.1135	12.367	0.015388	vapor
45.000	9.7660	0.45786	2.1841	404.50	425.68	1.7326	0.87329	1.0960	144.36	3.7963	12.586	0.015702	vapor
50.000	9.7660	0.44379	2.2533	409.25	431.10	1.7495	0.87043	1.0749	147.18	3.5254	12.803	0.016035	vapor
55.000	9.7660	0.43110	2.3196	413.94	436.44	1.7659	0.87025	1.0606	149.82	3.2900	13.017	0.016381	vapor
60.000	9.7660	0.41953	2.3836	418.60	441.72	1.7819	0.87187	1.0509	152.30	3.0827	13.229	0.016737	vapor

Problem 3

Let's try to calculate the Carnot efficiency of a gasoline engine by assuming that upon combustion all the higher heating value of the gas is used to heat the resulting product gases at constant volume.

Assume gasoline is pure octane (C8H18, Mw= 114), that air is 20% O2 80% N2 and that the amount of air needed is stoichiometric. You can assume that product gases are ideal gas and that their CV is 0.8 KJ/(kg K).

Solution:

ΔU calculation:

Gasoline HHV: **47** $\dfrac{MJ}{Kg}$ Basis of Calculation: **n** kmoles of gasoline (Octane)

$$\Delta U = Q + W = Q = HHV * m = 47 \left(\frac{MJ}{Kg}\right) * 114 \left(\frac{Kg}{Kmol}\right) * n(Kmol) = 5358 * n(MJ)$$

ΔT Calculation:
We can calculate the ΔT from the equation below:

$$\Delta U = f(T) \neq f(p,V) = m_{product} * C_v * \Delta T$$

ΔU and C_v are given so we need to find the amount of products we produce. From the stoichiometry of the reaction we have:

$$(n)C_8H_{18} + (n*12.5)O_2 \underset{\rightarrow}{} (n*8)CO_2 + (n*9)H_2O$$

Furthermore, we have the N_2 gas that is in the system. Considering the composition of air, number of moles of N_2 equals to $n*12.5*4$.
After the combustion, we will have CO_2, H_2O and N_2. From here we can calculate the mass of product as below:

$$m_{product} = n*8*44\left(\frac{Kg}{Kmol} \text{ for } CO_2\right) + n*9*18\left(\frac{Kg}{Kmol} \text{ for } H_{2O}\right) + n*50*28\left(\frac{Kg}{Kmol} \text{ for } N_2\right) = 1914*n(Kg)$$

Now we can calculate the ΔT:
$$\Delta T = \frac{\Delta U}{m_{product} * C_v}$$

$$5358 * n(MJ) = 1914 * n(Kg) * 0.8\left(\frac{KJ}{Kg.K}\right) * 10^{-3}\left(\frac{MJ}{KJ}\right) * \Delta T$$

$$\Delta T = 3499 \, (K)$$

In truth it's not that high. In part because the combustion is incomplete and because the Cv increases by about 50%. The actual ΔT is more like 1800 K.
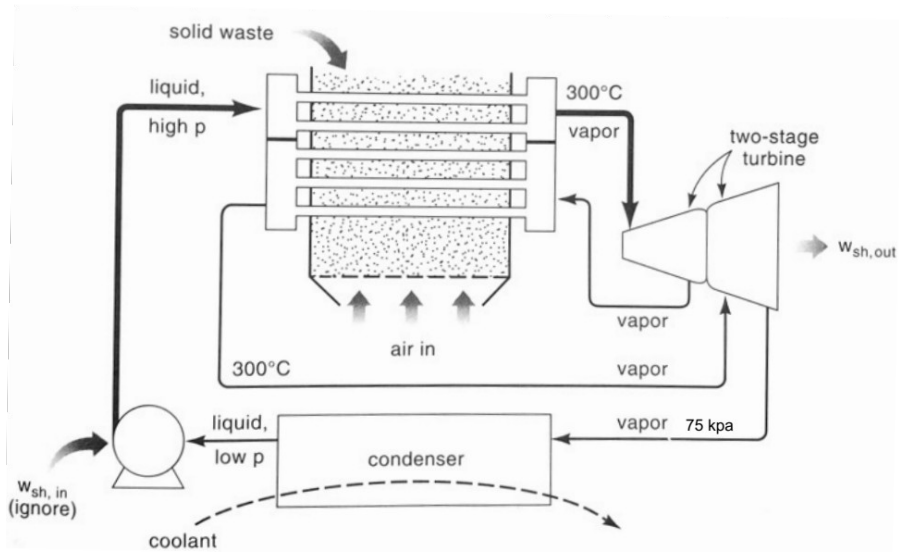
Efficiency calculation: (η)
By considering $T_C = 298.15 \, (K)$ we have:

$$\eta = \frac{T_H - T_C}{T_H} = \frac{\Delta T}{T_C + \Delta T} = \frac{3499}{298.15 + 3499} = 92\%$$

Problem 4

Imagine a power plant designed to burn EPFL's garbage. The temperature in the boiler cannot go higher than 300°C. The condenser can operate slightly below atmospheric pressure at 75 kPa. The working fluid is water, and because EPFL engineers are smart, they added a reheating cycle such that the setup looks like the following:
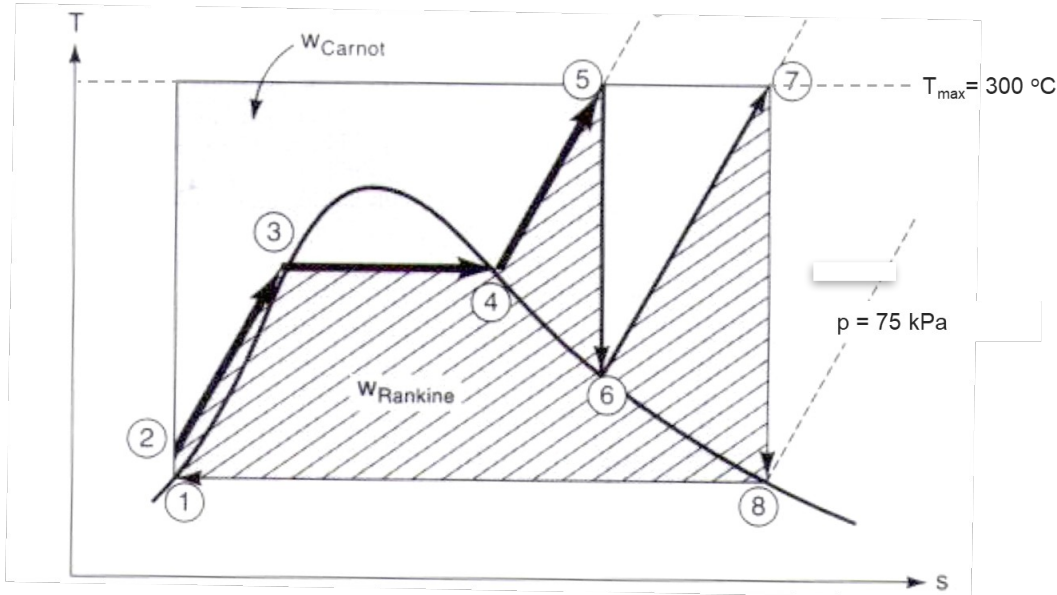

Can you calculate the efficiency? You will need steam tables, which you can find here: https://webbook.nist.gov/chemistry/fluid/

How does it compare to the Carnot efficiency?

Solution
In the solution, we will refer to the points on Figure bellow (identical to Fig 1.11 in the notes). We can sketch what we know :

To calculate the efficiency, note that we are producing work twice (processes 5-6 and 7-8) and reheating the working fluid twice (1-5 and 6-7). Therefore, the efficiency becomes:

$$\eta = \frac{|w|}{|q_c|} = \frac{|w_{5-6}| + |w_{7-8}|}{|q_{1-5}| + |q_{6-7}|} = \frac{|h_5 - h_6| + |h_7 - h_8|}{|h_5 - h_1| + |h_7 - h_6|}$$

If we start with point 1, we know that the condenser operates at 75 kPa and the working fluid is in a liquid state. From the steam tables we find that:

$$h_1 = 385 \frac{kJ}{kg} \quad s_1 = 1.22 \frac{kJ}{kg\,K}$$

Point 8 is at the state of saturated vapor. From the steam tables we can find that:

$$p_8 = 0.75\,¿$$
$$h_8 = 2659 \frac{kJ}{kg} \quad s_8 = 7.48 \frac{kJ}{kg\,K} \quad T_8 = 92\,°C$$

As we produce work in isentropic expansion from 7 to 8 that gives us:

$$s_8 = s_7 = 7.48 \frac{kJ}{kg\,K}$$

Additionally, maximum operating temperature of the boiler is $T_7 = 300\,°C$ which helps us find:

$p_7 = 4.8\, \bar{\iota}\quad h_7 = 3065\dfrac{kJ}{kg}$

Process 6-7 represents the heating of saturated vapor at constant pressure. Since we know the pressure at point 7, we can look up steam data for saturated vapor at $p_6 = 4.8\,\bar{\iota}$

$h_6 = 2746\dfrac{kJ}{kg}\ s_6 = 6.83\dfrac{kJ}{kg\ K}\ T_6 = 150.5\,°C$

Point 5 represents the starting point of the first isentropic expansion which gives

$s_5 = s_6 = 6.83\dfrac{kJ}{kg\ K}$

Just like in the case of the second heating, the temperature of the steam is the maximum temperature of the boiler:

$T_5 = 300\,°C\ h_5 = 3030\dfrac{kJ}{kg}\ p_5 = 17.7\,\bar{\iota}$

Now as we have gathered all the enthalpies, the efficiency becomes:

$$\eta = \frac{|w|}{|q_c|} = \frac{|w_{5-6}| + |w_{7-8}|}{|q_{1-5}| + |q_{6-7}|} = \frac{|h_5 - h_6| + |h_7 - h_8|}{|h_5 - h_1| + |h_7 - h_6|} = \frac{|3030 - 2746| + |3065 - 2659|}{|3030 - 385| + |3065 - 2746|} = \frac{690}{2964} = 23.3\%$$

Carnot's efficiency would for the given process would be:

$$\eta = \frac{T_H - T_C}{T_H} = \frac{(300 + 273)\,K - (92 + 273)\,K}{(300 + 273)\,K} = 36.3\%$$